TRINITY COLLEGE

Year 11 Mathematics Specialist Units 1, 2
Test 1 2020

Section 1 Calculator Free

Combinatorics and Vector Introduction

STUDENT'S NAME SOLUTIONS (PRESSER)

DATE: Wednesday 4 March **TIME:** 11 minutes **MARKS**: 11

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

(a) Calculate the number arrangements for the five letters; S W E E T [2]

$$\frac{5!}{2!} = \frac{5\times4\times3\times\cancel{2}\times\cancel{1}}{\cancel{2}\times\cancel{1}}$$

$$= 60$$

✓ numerator

✓ denominator

(b) Rewrite using factorial notation $n(n-1)(n-2)(n-3)$ [2]

$$= \frac{n!}{(n-4)!}$$

✓ numerator

✓ denominator

(c) Evaluate ${}^{20}C_{18}$ [2]

$$= \frac{20!}{2!\ 18!}$$

$$= \frac{1020\times19}{\cancel{2}\times1}$$

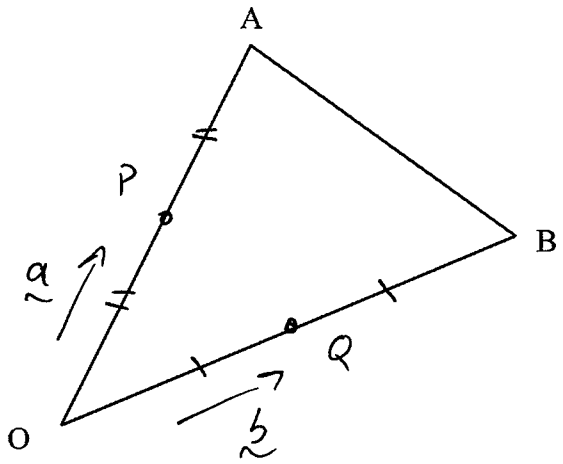
$$= 190$$

✓ expression

✓ answer

2. (5 marks)

Triangle OAB has $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.



(a) Determine a vector expression for $\overrightarrow{AB}$ [2]

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ ✓ step

$= -\underset{\sim}{a} + \underset{\sim}{b}$ ✓ answer

Point P and point Q divide $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in half respectively.

(b) Determine a vector expression for $\overrightarrow{PQ}$ [2]

$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$ ✓ step

$= -\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$ ✓ answer

(c) What conclusion can be drawn from your answers to part (a) and part (b) above? [1]

$\overrightarrow{PQ} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

$= \frac{1}{2}\overrightarrow{AB}$ ✓ parallel

$\therefore$ $\overrightarrow{PQ}$ is parallel to $\overrightarrow{AB}$

(it is also half the length)

TRINITY COLLEGE

Year 11 Mathematics Specialist Units 1, 2
Test 1 2020

Section 2 Calculator Assumed

Combinatorics and Vector Introduction

STUDENT'S NAME _____

DATE: Wednesday 4 March **TIME:** 39 minutes **MARKS:** 39

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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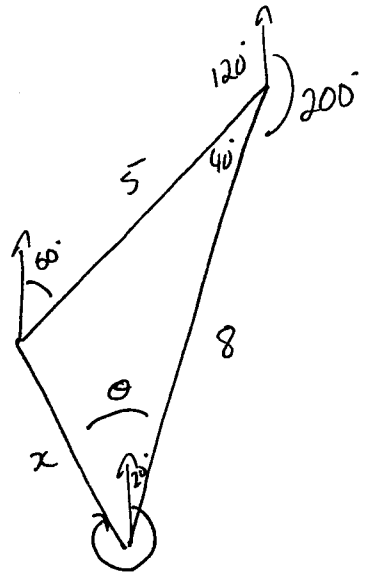
3. (5 marks)

Nathan sets of on an orienteering event. He walks 5 km on a bearing of 060°*T* and then 8 km on a bearing of 200°*T* . Determine the vector Nathan must set to return to his starting position.

✓ diagram
✓ cosine rule
✓ distance
✓ angle
✓ bearing



$x^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 40°$

$\Rightarrow x = 5.26$ km

$\frac{\sin\theta}{5} = \frac{\sin 40°}{5.26}$

$\Rightarrow \theta = 37.62°$

∴ Nathan's vector is 5.26 km on bearing of 342.38°T (~ 342°T)

4. (9 marks)

A group of four males and three females are selected from a group of 10 males and a group of 8 females, and arranged for a photograph. Determine the number of ways they can be arranged if:

(a) there are no restrictions. [1]

$^{10}C_4\ ^{8}C_3 \times 7! = 59\,270\,400$ ✓ solution

(b) three males, John, Paul and Patrick, must be next to each other. [3]

$^{3}C_3\ ^{7}C_1\ ^{8}C_3\ 5!\ 3! = 282\,240$

✓ choose
✓ $5!$
✓ $3!$

(c) two females, Amanda and Betty, must not be next to each other. [3]

all − Amanda & Betty together

$^{10}C_4\ ^{8}C_3\ 7! - {}^{2}C_2\ ^{10}C_4\ ^{6}C_1\ 6!\ 2!$

$= 59\,270\,400 - 1814\,400$

$= 57\,456\,000$

✓ all − together
✓ Amanda & Betty
✓ answer

(d) no two males are to be together. [2]

M F M F M F M

$^{10}C_4\ ^{8}C_3\ 4!\ 3!$

$= 1\,693\,440$

✓ choose
✓ $4!\ 3!$

5. (8 marks)

Four digit PINs are to be formed using the digits 0 to 9 inclusive. Determine how many four digit PINs are possible if:

(a) digits can be repeated. [1]

$10^4 = 10000$ ✓ answer

(b) digits cannot be repeated and the PIN must be greater than 4000. [2]

$6 \times 9 \times 8 \times 7$

$= 3024$

OR $^6C_1 \, ^9C_3 \times 3!$

_ _ _ _

4
5
6
7
8
9

✓ $6 \times 9 \times 8 \times 7$ choose
✓ answer or arrange

(c) digits cannot be repeated and the PIN must be even. [2]

$5 \times 9 \times 8 \times 7$

$= 2520$

OR $^5C_1 \, ^9C_3 \times 3!$

_ _ _ _

2
4
6
8
0

✓ $5 \times 9 \times 8 \times 7$
✓ answer

(d) digits cannot be repeated and the PIN must be greater than 4000 or even. [3]

$n(b \cup c) = n(b) + n(c) - n(b \cap c)$

$= 3024 + 2520 - (3 \times 8 \times 7 \times 4 + 3 \times 8 \times 7 \times 5)$

4 6 8 _ _ _ 5 7 9 _ _ _

✓ inclusion/exclusion
✓ $n(b \cap c)$
✓ answer

$= 3024 + 2520 - (672 + 840)$

$= 4032$

6. (8 marks)

(a) Determine how many integers between 1 and 10 000 inclusive are divisible by 2, 3 or 5. [5]

$n(2) = 5000$
$n(3) = 3333$
$n(5) = 2000$

✓ $n(a)$

$n(2 \cap 3) = n(6) = 1666$
$n(2 \cap 5) = n(10) = 1000$
$n(3 \cap 5) = n(15) = 666$

✓ $n(a \cap b)$

$n(2 \cap 3 \cap 5) = n(30) = 333$

✓ $n(a \cap b \cap c)$

$n(2 \cup 3 \cup 5) = 5000 + 3333 + 2000 - 1666 - 1000 - 666 + 333$

✓ inclusion exclusion

$= 7334$

✓ answer

(b) 51 different integers are chosen from the integers between 1 and 100 inclusive. Prove that two of the chosen integers are consecutive. [3]

✓ setup

Let the pigeon holes be consecutive integers

ie $\{1,2\}$ $\{3,4\}$ ... $\{99,100\}$

There are 50 pigeon holes.

✓ argument

Therefore, by the P.H.P, there must be at least one set of consecutive integers (pigeon hole) that contains two integers if we have 51 integers.

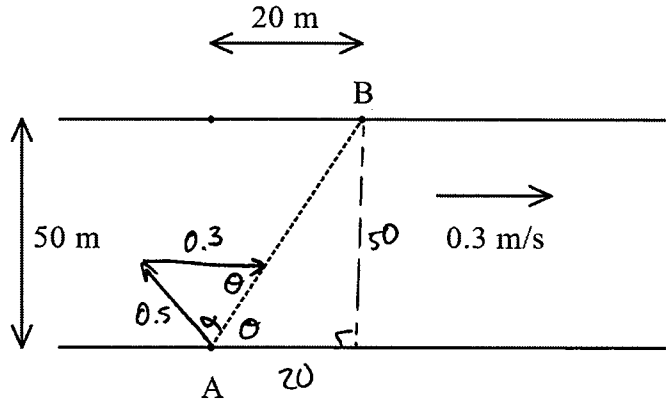
✓ conclusion

Therefore, by the P.H.P, two integers must be consecutive.

7. (9 marks)

Anthony wishes to paddle his canoe across a river that is 50 m, from point A to point B which is 20 m downstream. Anthony can maintain a constant 5 km/h in still water. However, the river is flowing at 3 km/h.

0.3 m/s 0.5 m/s

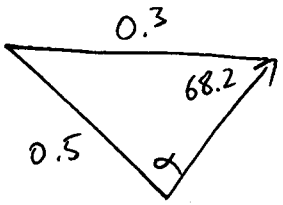


(a) Determine the direction Anthony must paddle to reach B directly from A. [3]

✓ 68.2° $\tan\theta = \frac{50}{20}$

✓ α $\theta = 68.2°$

✓ angle to bank



$\frac{\sin\alpha}{0.3} = \frac{\sin 68.2}{0.5}$

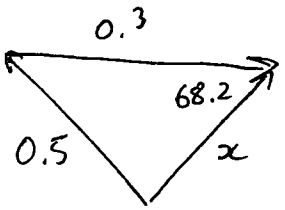
$\alpha = 33.85°$

∴ at ≈ 78° to bank

(b) Determine the resultant velocity. [2]

✓ cosine rule

✓ answer



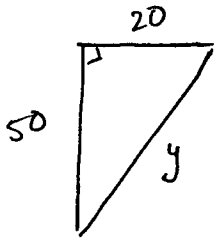
$0.5^2 = 0.3^2 + x^2 - 2\times0.3\times x\times\cos 68.2°$

$\Rightarrow x = 0.53$ m/s

(c) Determine the time it takes Anthony to reach B. [2]

✓ distance

✓ time



$y^2 = 50^2 + 20^2$

$\Rightarrow y = \sqrt{2900}$

$v = \frac{d}{t}$ $t = \frac{\sqrt{2900}}{0.53}$

$= 102.2$ sec

(d) Anthony now wishes to return directly to *A* from *B*. Will the return trip take more time, the same time, or less time than your answer in part (b)? Explain. [2]

✓ more

✓ reason

It will take more time.

This is because he now needs to paddle upstream (against the current) and his resultant velocity will be less.